Relative representation theory – Harmonic analysis over spherical varieties

A. Aizenbud

Weizmann Institute of Science

http://aizenbud.org
Observation

*Representation theory of G*

\[ \updownarrow \]

*Harmonic analysis on G w.r.t. the two sided action of G \times G*

Conclusion

Let G act on a space X. One can consider harmonic analysis over X (i.e. the study of the G representation $F(X)$) as a generalization of representation theory.

Example

Schur's lemma is analogous to the Gelfand property:

$\forall \pi \in \text{irr}(G) : \langle F(X), \pi \rangle \leq 1$

Conjecture

Let G be a reductive algebraic group scheme and X be a spherical G space (i.e. over any algebraically closed field, the Borel acts with finitely many orbits on X).

Then $\sup F$ is a finite or local field ($\sup \rho \in \text{irr}(G(F)) \langle F(X), \rho \rangle < \infty$).
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\sup_{F \text{ is a finite or local field}} \left( \sup_{\rho \in \text{irr}(G(F))} \langle F(X), \rho \rangle \right) < \infty.
\]