Cohen macauly property in representation theory

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Some Phenomena in Harmonic Analysis

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Let $K < G$ be a compact open subgroup. Sometimes $\mathcal{S}(G/H)^K$ is free over the center of the Hecke algebra $\mathcal{H}(G,K)$.

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Definition

In this case M is called a Cohen-Macaulay module over A.

The Main Conjecture

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Let X be a (symmetric) G-space. Then
$\mathcal{S}(X)$ is a Cohen-Macaulay object in the category of smooth G-modules.