Theorem 1 *Hironaka. Let X be an affine Nash manifold. Then there exists a compact affine nonsingular algebraic variety $Y \supset X$ s.t. $Y = X \dot\cup D \dot\cup U$ where X and U are open and $D = \bigcup_{i=1}^{k} D_i$ where $D_i \subset Y$ are closed Nash submanifolds of codimension 1 and all the intersections are normal, i.e. every $y \in Y$ has a neighborhood V with a diffeomorphism $\phi : V \to \mathbb{R}^n$ s.t. $\phi(D_i \cap V)$ is either a coordinate hyperplane or empty.*

Theorem 2 *Local triviality of Nash manifolds. Any Nash manifold can be covered by finite number of open submanifolds Nash diffeomorphic to $\mathbb{R}^n$.*

Theorem 3 *Let X be a Nash manifold. Then $H^i(X)$,$H^i_c(X)$,$H_i(X)$ are finite dimensional.*

Theorem 4 *Let X be an affine Nash manifold. Consider the De-Rham complex of X with compactly supported coefficients*

$$DR_c(X) : 0 \to C_c^\infty(X, \Omega_X^0) \to ... \to C_c^\infty(X, \Omega_X^n) \to 0,$$

the De-Rham complex of X with Schwartz coefficients

$$DR_S(X) : 0 \to S(X, \Omega_X^0) \to ... \to S(X, \Omega_X^n) \to 0,$$

and the natural map $i : DR_c(X) \to DR_S(X)$.
Then i is a quasiisomorphism, i.e. it induces an isomorphism on the cohomologies.

Theorem 5 *Let X be an affine Nash manifold. Consider the De-Rham complex of X with coefficients in classical generalized functions, i.e. functionals on compactly supported densities*

$$DR_G(X) : 0 \to C^{-\infty}(X, \Omega_X^0) \to ... \to C^{-\infty}(X, \Omega_X^n) \to 0,$$

the De-Rham complex of X with coefficients in generalized Schwartz functions

$$DR_{GS}(X) : 0 \to C^{-\infty}(X, \Omega_X^0) \to ... \to C^{-\infty}(X, \Omega_X^n) \to 0,$$

and the natural map $i : DR_{GS}(X) \to DR_G(X)$. Then i is a quasiisomorphism.

Theorem 6 *Let X be an affine Nash manifold of dimension n. Then*
$H^i(DR_{GS}(X)) \cong H^i(X)$
$H^i(DR_S(X)) \cong H_c^i(X)$
$H^i(TDR_S(X)) \cong H_i(X)$
and the standard pairing between $S(X, T\Omega_X^{n-i})$ and $GS(X, \Omega_X^i)$ gives an isomorphism between $H^i(DR_{GS}(X))$ and $(H^i(TDR_S(X)))^*$.

Definition 1 Let X and Y be Nash manifolds and let E and F be Nash bundles over them. We define integration by fibers $IF : S(X \times Y, E \boxtimes F) \times GS(Y, \widetilde{F}) \to \Gamma(X, E)$ (where $\Gamma(X, E)$ is the space of all global sections of E over X) by $IF(\xi, \eta)(x) = \eta(\xi|_{\{x\} \times Y})$.

Proposition 7 *$Im(IF) \subset S(X, E)$, and $IF : S(X \times Y, E \boxtimes F) \times GS(Y, \widetilde{F}) \to S(X, E)$ is continuous.*

Definition 2 Let X and Y be Nash manifolds and let E and F be Nash bundles over them. We define $IF' : GS(X \times Y, E \boxtimes F) \times S(Y, \widetilde{F}) \to GS(X, E)$ by $IF'(\xi, \eta)(f) = \xi(\eta \boxtimes f)$.

Theorem 8 *Let X be a Nash manifold and Y be an affine Nash manifold of dimensions m and n correspondingly. Denote $F = X \times Y$. Identifying $T\Omega^i_{F \to X}$ with $(X \times \mathbb{R}) \boxtimes T\Omega^i_Y$ (where $X \times \mathbb{R}$ is interpreted as the trivial bundle on X) and Ω^{n-i}_Y with $\widetilde{T\Omega^i_Y}$ we get bilinear form $IF : S(F, T\Omega^i_{F \to X}) \times GS(Y, \Omega^{n-i}_Y) \to S(X)$. Then IF induces a non degenerate pairing between $H^i(TDR_S(F \to X))$ and $H^{n-i}(Y)$ valued in $S(X)$ i.e. gives an isomorphism between $H^i(TDR_S(F \to X))$ and $H^{n-i}(Y)^* \otimes S(X)$.*

Theorem 9 *Let X be a Nash manifold and Y be an affine Nash manifold of dimensions m and n correspondingly. Denote $F = X \times Y$. Then the bilinear form $IF' : GS(F, \Omega^i_{F \to X}) \times S(Y, T\Omega^{n-i}_Y) \to GS(X)$ gives an isomorphism between $H^i(DR_{GS}(F \to X))$ and $H_{n-i}(Y)^* \otimes GS(X)$.*

Theorem 10 *Let $p : F \to X$ be a Nash locally trivial fibration and E be a Nash bundle over X. Then $H^k(DR_S^E(F \to X)) \cong S(X, H_c^k(F \to X) \otimes E)$.*

Theorem 11 *Let $p : F \to X$ be a Nash locally trivial fibration and E be a Nash bundle over X. Then $H^k(DR_{GS}^E(F \to X)) \cong GS(X, H^k(F \to X) \otimes E)$.*

Definition 3 Let $f : X \to Y$ be a Nash map of Nash manifolds. It is called a *Nash locally trivial fibration* if there exist a Nash manifold M and surjective submersive Nash map $g : M \to Y$ s.t. the basechange $h : X \underset{Y}{\times} M \to M$ is trivializable, i.e. there exists a Nash manifold Z and an isomorphism $k : X \underset{Y}{\times} M \to M \times Z$ s.t. $\pi \circ k = h$ where $\pi : M \times Z \to M$ is the standard projection.

Theorem 12 *Let M and N be Nash manifolds and $s : M \to N$ be a surjective submersive Nash map. Then locally it has a Nash section, i.e. there exists a finite open cover $N = \bigcup_{i=1}^{k} U_i$ s.t. s has a Nash section on each U_i.*

Proposition 13 *Let M and N be Nash manifolds and $f : M \to N$ be a Nash submersion. Let $L \subset N$ be a Nash submanifold and $s : L \to M$ be a section of f. Then there exist a finite open Nash cover $L \subset \bigcup_{i=1}^{n} U_i$ and sections $s_i : U_i \to M$ of f s.t. $s|_{L \cap U_i} = s_i|_{L \cap U_i}$.*

Theorem 14 *Any semi-algebraic surjection $f : X \to Y$ of semi-algebraic sets has a semi-algebraic section.*

Theorem 15 *Let $f : M \to N$ be a semi-algebraic map of Nash manifolds. Then there exists a finite stratification of M by Nash manifolds $M = \dot{\bigcup}_{i=1}^{k} M_i$ s.t. $f|_{M_i}$ is Nash.*

Definition 4 Let $\mathfrak{g}$ be a Lie algebra of dimension n. Let ρ be its representation. Define $H^i(\mathfrak{g}, \rho)$ to be the cohomologies of the complex:

$$C(\mathfrak{g}, \rho) : 0 \xrightarrow{d} \rho \xrightarrow{d} \mathfrak{g}^* \otimes \rho \xrightarrow{d} (\mathfrak{g}^*)^{\wedge 2} \otimes \rho \xrightarrow{d} ... \xrightarrow{d} (\mathfrak{g}^*)^{\wedge n} \otimes \rho \xrightarrow{d} 0$$

with the differential defined by

$$d\omega(x_1, ..., x_{n+1}) = \sum_{i=1}^{n+1} (-1)^i \rho(x_i)\omega(x_1, ..., x_{i-1}, x_{i+1}, ..., x_{n+1}) +$$
$$+ \sum_{i<j} (-1)^{i+j} \omega([x_i, x_j], x_1, ..., x_{i-1}, x_{i+1}, ..., x_{j-1}, x_{j+1}, ..., x_{n+1})$$

where we interpret $(\mathfrak{g})^{\wedge k} \otimes \rho$ as anti-symmetric ρ-valued k-forms on $\mathfrak{g}$.

Remark 16 *$H^i(\mathfrak{g}, \rho)$ is the i-th derived functor of the functor $\rho \mapsto \rho^{\mathfrak{g}}$.*

Theorem 17 *Let G be a Nash group. Let X be a Nash G-manifold and $E \to X$ a Nash G-equivariant bundle. Let Y be a strictly simple Nash G-manifold. Suppose Y and G are cohomologically trivial (i.e. all their cohomologies except H^0 vanish and $H^0 = \mathbb{R}$) and affine. Denote $F = X \times Y$. Note that the bundle $E \boxtimes \Omega_Y^i$ has Nash G-equivariant structure given by diagonal action. Hence the relative De-Rham complex $DR_{GS}^E(F \to X)$ is a complex of representations of $\mathfrak{g}$. Then $H^i(\mathfrak{g}, GS(X, E)) = H^i((DR_{GS}^E(F \to X))^{\mathfrak{g}})$.*

Proposition 18 *Let G be a connected Nash group and F be a Nash G manifold with strictly simple action. Denote $X := G \setminus F$ and let $E \to X$ be a Nash bundle. Then $(GS(F, \pi^*(E)))^{\mathfrak{g}} \cong GS(X, E)$ where $\pi : F \to X$ is the standard projection.*

Corollary 19 *Let G be a Nash group and X be a transitive Nash G manifold. Let $x \in X$ and denote $H := stab_G(x)$. Consider the diagonal action of G on $X \times G$. Let $E \to X \times G$ be a G equivariant Nash bundle. Then $GS(X \times G, E)^{\mathfrak{g}} \cong GS(\{x\} \times G, E|_{\{x\}\times G})^{\mathfrak{h}}$.*

Theorem 20 *Shapiro lemma. Let G be a Nash group and X be a transitive Nash G manifold. Let $x \in X$ and denote $H := stab_G(x)$. Let $E \to X$ be a G equivariant Nash bundle. Let V be the fiber of E in x. Suppose G and H are cohomologically trivial. Then $H^i(\mathfrak{g}, GS(X, E)) \cong H^i(\mathfrak{h}, V)$.*