WF-holonomicity of constructible distributions on non-archimedean local fields

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- ☹ No actin of differential operators on distributions

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☹ WF-holonomicity is not stable under Fourier transform

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It is easy to see that WL is 1-1.

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- *Constructible distributions are smooth almost everywhere.*

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Main ingredients of the proof:

- *Regularization:* a constructible distribution can be extended from an open set
- *Resolution of singularities in the constructible (in fact, definable) setting.*
- *Key-Lemma:* a smooth constructible function on an open set can be extended to an holonomic constructible distribution.

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- adding 1 point does not affect WF-holonomicity